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# Edge degree conditions for subpancyclicity in line graphs

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**Abstract**

In this paper, two best possible edge degree conditions are given for the line graph $L(G)$ of a graph $G$ with girth at least 4 or 5 to be subpancyclic, i.e., $L(G)$ contains a cycle of length $k$, for each $k$ between 3 and the circumference of $L(G)$. In [5] the following conjecture is made: If $G$ is a graph such that the degree sum of any pair of adjacent vertices in $G$ is greater than $(\sqrt{8n+1}+1)/2$, then the line graph $L(G)$ of $G$ is pancyclic whenever $L(G)$ is Hamiltonian, unless $G$ is isomorphic to $C_4$, $C_5$, or the Petersen graph. Our results show that the conjecture is true for those graphs of order $n \geqslant 72$ with girth at least 4. 

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*Keywords:* Line graph; Hamiltonian graph; Subpancyclic graph

## 1. Introduction

We use [1] for terminology and notation not defined here and consider finite simple graphs only. Set $\lambda(G)=\{k \mid G$ contains a cycle of length $k\}$. A graph $G$ is *pancyclic* if $\lambda(G)=[3,|V(G)|]$, i.e., $\lambda(G)$ contains all integers in $[3,|V(G)|]$. Similarly, a graph $G$ is called *subpancyclic* if $\lambda(G)=[3,\mathrm{cr}(G)]$ where $\mathrm{cr}(G)$ denotes the *circumference* of a graph $G$, i.e. the length of a longest cycle of $G$. Let $g(G)$ denote the *girth* of a graph, i.e., the length of a shortest cycle of $G$. The degree of an edge $e=uv\in E(G)$, denoted by $d(e)$, is defined to be $d(u)+d(v)$. The *minimum edge degree* of a graph $G$, denoted by $\delta'(G)$, is defined to be $\min\{d(u)+d(v) \mid uv\in E(G)\}$.

A natural question is the following: how large should be the minimum degree $\delta(G)$ of a graph $G$ be in order that the Hamiltonian line graph $L(G)$ is guaranteed to be pancyclic?

From results in [4] it follows that there exists a constant $A$ such that if $L(G)$ is Hamiltonian and $\delta(G)>A\cdot n^{1/3}$, then $L(G)$ is pancyclic. Also, there exists a constant $B$


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and an infinite family of graphs $G$ such that $\delta(G) > B \cdot n^{1/3}$ and $L(G)$ is Hamiltonian but not pancyclic.

Here we consider a similar question concerning edge degree.

Our main results are the following.

**Theorem 1.** *Let $G$ be a graph of order $n$ ($n \geqslant 72$), girth at least 5, and with $\delta'(G) > \sqrt{2n+1}$ (or $\delta'^2(G) - 1 > 2n$). Then $L(G)$ is subpancyclic and the result is best possible.*

**Theorem 2.** *Let $G$ be a graph of order $n$ ($n \geqslant 72$), girth at least 4, and with $\delta'^2(G) - \delta'(G) > 2n$. Then $L(G)$ is subpancyclic and the result is best possible.*

In general, results involving degree sums are direct derivatives from results involving the minimum degree of the graph. Theorems 1 and 2 show an exception to this rule (compare the results in [4] mentioned above).

The main theorems in the paper deal with graphs with girth at least 5 (Theorem 1) and girth at least 4 (Theorem 2). Since the necessary degree condition for subpancyclicity in Theorem 1 is a little weaker than the degree condition in Theorem 2, an obvious question is if one can expect an even weaker condition for graphs with girth at least $k$ for some $k \geqslant 6$. But, in fact, the extremal graphs given at the end of the proof of Theorem 1 can be chosen to have arbitrarily large girth. This shows that even for graphs with large girth the degree condition in Theorem 1 is best possible.

## 2. Proofs of the main theorems

Before proving Theorems 1 and 2 we introduce some additional terminology and notation, and state a number of preliminary results.

By a *circuit* of a graph $G$ we will mean an Eulerian subgraph of $G$, i.e., a connected subgraph in which every vertex has even degree. Note that by this definition (the trivial subgraph induced by) a single vertex is also a circuit. If $\mathscr{C}$ is a circuit of $G$, then $\bar{E}(\mathscr{C})$ denotes the set of edges of $G$ incident with at least one vertex of $\mathscr{C}$, we write $\varepsilon(\mathscr{C})$ for $|E(\mathscr{C})|$ and $\bar{\varepsilon}(\mathscr{C})$ for $|\bar{E}(\mathscr{C})|$. Let $\mathscr{C}_m$ denote a cycle of length $m$.

Harary and Nash–Williams [3] characterized Hamiltonian line graphs.

**Theorem 3** (Harary and Nash–Williams [3]). *The line graph $L(G)$ of a graph $G$ is Hamiltonian if and only if $G$ contains a circuit $\mathscr{C}$ such that $\bar{\varepsilon}(\mathscr{C}) = |E(G)| \geqslant 3$.*

From Theorem 3 one easily prove a more general result (see e.g., [2]).

**Theorem 4.** *The line graph $L(G)$ of a graph $G$ contains a cycle of length $k \geqslant 3$ if and only if $G$ contains a circuit $\mathscr{C}$ such that $\varepsilon(\mathscr{C}) \leqslant k \leqslant \bar{\varepsilon}(\mathscr{C})$.*

A key lemma for our proof of Theorems 1 and 2 is the following.

**Lemma 5.** *Let $G$ be a graph of order $n$, girth at least 4, and with minimum edge degree $\delta' \geqslant 8$ such that $L(G)$ contains $\mathscr{C}_{m+1}$ but not $\mathscr{C}_m$. Then*

$$m \leqslant \frac{4n - \delta'}{\delta'}.$$

**Proof.** Let $G$ satisfy the hypothesis of the lemma. By Theorem 4, $G$ contains a circuit $\mathscr{C}$ with $\varepsilon(\mathscr{C}) \leqslant m + 1 \leqslant \bar{\varepsilon}(\mathscr{C})$. In fact, $\varepsilon(\mathscr{C}) = m + 1$, otherwise $L(G)$ contains $\mathscr{C}_m$. By Theorem 4, $\varepsilon(\mathscr{C}) \geqslant \Delta(G) + 2 \geqslant (\delta'(G) + 4)/2 \geqslant 6$. Since $\mathscr{C}$ is a circuit, there exist edge disjoint cycles $D_1, \ldots, D_r$ such that

$$\mathscr{C} = \bigcup_{i=1}^{r} D_i.$$

We distinguish the following cases.

*Case* 1: $r = 1$. Then $\mathscr{C}$ is a cycle of length $m + 1$.

*Case* 1.1: *$\mathscr{C}$ has a chord.* Let $\mathscr{C}'$ be a longest cycle among all cycles that contain exactly one chord of $\mathscr{C}$ while the remaining edges belong to $\mathscr{C}$.

In $\sum_{e \in E(\mathscr{C}')} d(e)$, every edge in $\bar{E}(\mathscr{C}')$ is counted at most four times. Hence,

$$\bar{\varepsilon}(\mathscr{C}') \geqslant \varepsilon(\mathscr{C}')\delta'/4 > \varepsilon(\mathscr{C})\delta'/8 = (m + 1)\delta'/8 \geqslant m + 1.$$

On the other hand, $\bar{\varepsilon}(\mathscr{C}') \leqslant m$. Thus, $L(G)$ contains $\mathscr{C}_m$, a contradiction.

*Case* 1.2: *$\mathscr{C}$ has no chord.* Since $\delta' \geqslant 8$, $\mathscr{C}$ cannot be a Hamilton cycle of $G$. Let $u$ be a vertex in $V(G) \backslash V(\mathscr{C})$. If $u$ is adjacent to at least three vertices of $\mathscr{C}$, then $G$ contains a cycle $\mathscr{C}'$ with $\varepsilon(\mathscr{C})/2 < \varepsilon(\mathscr{C}') \leqslant m$ and we obtain a contradiction as in Case 1.1. Hence, $u$ is adjacent to at most two vertices of $\mathscr{C}$. Defining $p$ as the number of edges incident with exactly one vertex of $\mathscr{C}$, we thus have

$$p \leqslant 2\,|V(G) \backslash V(\mathscr{C})| = 2(n - m - 1).$$

On the other hand, since $G$ has no chords,

$$p = \sum_{e \in E(\mathscr{C})} (d(e) - 4))/2 \geqslant (m + 1)(\delta' - 4)/2.$$

It follows that $(m + 1)(\delta' - 4)/2 \leqslant 2(n - m - 1)$ or, equivalently,

$$m \leqslant \frac{4n - \delta'}{\delta'}.$$

*Case* 2: $r \geqslant 2$. Let $H$ be the graph with $V(H) = \{D_1, \ldots, D_r\}$ and $D_iD_j \in E(H)$ iff $V(D_i) \cap V(D_j) \neq \emptyset\,(i \neq j)$. Since $H$ is connected, at least two vertices of $H$ are not cut vertices of $H$. Equivalently, there are at least two values of $j$ for which $\bigcup_{1 \leqslant i \leqslant r, i \neq j} D_i$ is a connected subgraph of $G$, and hence a circuit of $G$.

Assume, without loss of generality, that $\mathscr{C}' = \bigcup_{i=2}^{r} D_i$ and $\mathscr{C}'' = D_1 \cup \bigcup_{i=3}^{r} D_i$ are circuits of $G$. We have

$$\begin{aligned}\bar{\varepsilon}(\mathscr{C}') &\geqslant |E(\mathscr{C}')| + |E(D_1) \cap \bar{E}(\mathscr{C}')| + |E(D_2 - V(\mathscr{C}''))|(\delta' - 4)/4 \\ &= |E(\mathscr{C})| - |E(D_1 - V(\mathscr{C}'))| + |E(D_2 - V(\mathscr{C}''))|(\delta' - 4)/4.\end{aligned}$$

On the other hand, since $L(G)$ does not contain $\mathscr{C}_m$,

$$\bar{\varepsilon}(\mathscr{C}') \leqslant m - 1 = \varepsilon(\mathscr{C}) - 2.$$

It follows that $|E(D_1 - V(\mathscr{C}'))| \geqslant |E(D_2 - V(\mathscr{C}''))|(\delta' - 4)/4 + 2$ and hence, since $\delta' \geqslant 8$,

$$|E(D_1 - V(\mathscr{C}'))| > |E(D_2 - V(\mathscr{C}''))|.$$

But then by symmetry we also have

$$|E(D_2 - V(\mathscr{C}''))| > |E(D_1 - V(\mathscr{C}'))|.$$

This contradiction completes the proof. □

In a way similar to the proof of Lemma 5, one easily proves the following.

**Lemma 5′.** *Let $G$ be a graph of order $n$, girth at least 5, and with minimum edge degree $\delta' \geqslant 8$ such that $L(G)$ contains $\mathscr{C}_{m+1}$ but not $\mathscr{C}_m$. Then*

$$m \leqslant \frac{2n - \delta' + 2}{\delta' - 2}.$$

The proofs of our main results also need the following lemmas.

**Lemma 6.** *Let $G$ be a graph of order $n$ with $\delta' > \sqrt{2n+1}$ ($n \geqslant 72$). Then $g(G) \leqslant \Delta(G) + 1$.*

**Proof.** Since $n \geqslant 72$, we have $\delta' \geqslant 13$. Let $G$ satisfy the hypothesis of the lemma and $\mathscr{C}$ be a shortest cycle of $G$.

We distinguish the following two cases.

*Case* 1: $\delta(G) = 1$. Obviously, $\Delta(G) \geqslant \delta' - 1$. Set $E_1(\mathscr{C}) = \{e = uv \in E(\mathscr{C}) \mid \text{either } u \text{ or } v \text{ is adjacent to an isolated vertex in } G - V(\mathscr{C})\}$, $E_2(\mathscr{C}) = E(\mathscr{C}) \backslash E_1(\mathscr{C})$ and $V_2(\mathscr{C}) = V(G[E_2(\mathscr{C})])$.

We have the following claim.

$$d(e) \geqslant (\delta' - 1) + 2 = \delta' + 1 \quad \text{for any edge } e \in E_1(\mathscr{C}). \tag{1}$$

Assuming $\varepsilon(\mathscr{C}) \geqslant \Delta(\mathscr{C}) + 2$, we thus have $\varepsilon(\mathscr{C}) \geqslant \delta' + 1 \geqslant 14$, implying that

$$|\{uv \in E(G)\colon v \in V(\mathscr{C})\}| \leqslant 1 \quad \text{for any } u \in V(G) \backslash V(\mathscr{C}). \tag{2}$$

By the choice of $\mathscr{C}$, we obtain that for any vertex $x \in V_2(\mathscr{C})$, there exists a set $\{p_i\}$ of $d(x) - 2$ paths of length 2 such that $V(p_i) \cap V(\mathscr{C}) = \{x\}$ and $V(p_i) \cap V(p_j) = \{x\}$ $(i \neq j)$.

Defining $P_1$ as the set of such paths, we thus obtain that any pair of paths in $P_1$ have at most one common vertex in $G$ and

$$|P_1| \geqslant \sum_{e \in E_2(\mathscr{C})} (d(e) - 4))/2 \geqslant |E_2(\mathscr{C})|/2. \tag{3}$$

Using (1)–(3) and $\delta' > \sqrt{2n+1}$, we obtain

$$\begin{aligned} n &\geqslant \left(\sum_{e\in E(\mathscr{C})} (d(e)-4)\right) \Big/ 2 + \varepsilon(\mathscr{C}) + |P_1| \\ &\geqslant \left(\sum_{e\in E_1(\mathscr{C})} d(e) + \sum_{e\in E_2(\mathscr{C})} d(e)\right) \Big/ 2 + |P_1| - \varepsilon(\mathscr{C}) \\ &\geqslant ((\delta'+1)|E_1(\mathscr{C})| + \delta'|E_2(\mathscr{C})|)/2 + |E_2(\mathscr{C})|/2 - \varepsilon(\mathscr{C}) \\ &= (\delta'+1)(|E_1(\mathscr{C})| + |E_2(\mathscr{C})|)/2 - \varepsilon(\mathscr{C}) \\ &= \varepsilon(\mathscr{C})(\delta'-1)/2 \geqslant (\delta'+1)(\delta'-1)/2 > n, \end{aligned}$$

a contradiction.

*Case* 2: $\delta(G) > 1$. Assuming $\varepsilon(\mathscr{C}) \geqslant \Delta(G) + 2$, we thus have $\varepsilon(\mathscr{C}) \geqslant (\delta'+4)/2 \geqslant \frac{17}{2}$, implying that $\varepsilon(\mathscr{C}) \geqslant 9$. By the choice of $\mathscr{C}$ and $\delta(G) > 1$, we have that for any vertex $x \in V(\mathscr{C})$, there exists a set $\{p_i\}$ of $d(x) - 2$ paths of length 2 such that $V(p_i) \cap V(\mathscr{C}) = \{x\}$ and $V(p_i) \cap V(p_j) = \{x\}$ $(i \neq j)$.

Defining $P_2$ as the set of such paths, we thus obtain that any pair of paths in $P_2$ have at most one common vertex in $G$ and

$|P_2| \geqslant \sum_{u\in V(\mathscr{C})} (d(u) - 2)$, implying that

$$\begin{aligned} n &\geqslant 2\sum_{u\in V(\mathscr{C})} (d(u)-2) + \varepsilon(\mathscr{C}) = 2\left(\sum_{e\in E_1(\mathscr{C})} (d(e)-4)\right) \Big/ 2 + \varepsilon(\mathscr{C}) \\ &\geqslant ((\delta'-4)+1)\varepsilon(\mathscr{C}) \geqslant (\delta'-3)(\delta'+4)/2 > n, \end{aligned}$$

a contradiction. This completes the proof. □

**Lemma 7.** *Let $G$ be a graph of order $n$, girth at least 5, and with $g(G)(\delta'-2)^2 + 2(\delta'-2) \geqslant 4n$. Then*

$$\lambda(L(G)) = [g(G), \mathrm{cr}(L(G))].$$

**Proof.** Let $G$ satisfy the hypothesis of the lemma and $\mathscr{C}$ be a shortest cycle of $G$, i.e., $\varepsilon(\mathscr{C}) = g(G)$. We have

$$\begin{aligned} \bar{\varepsilon}(\mathscr{C}) &= \left(\sum_{e\in E(\mathscr{C})} (d(e)-4)\right) \Big/ 2 + \varepsilon(\mathscr{C}) \\ &= \left(\sum_{e\in E(\mathscr{C})} d(e)\right) \Big/ 2 - \varepsilon(\mathscr{C}) \\ &\geqslant (\delta'-2)\varepsilon(\mathscr{C})/2 \\ &= g(G)(\delta'-2)/2. \end{aligned} \tag{4}$$

By $g(G)(\delta'-2)^2+2(\delta'-2)\geqslant 4n$,

$$g(G)(\delta'-2)/2\geqslant 2n/(\delta'-2)-1. \tag{5}$$

Using (4) and (5), Lemma 5′ and Theorem 4, we obtain $\lambda(L(G))=[g(G),\mathrm{cr}(L(G))]$, which completes the proof of Lemma 7. □

**Proof of Theorem 1.** Let $G$ satisfy the hypothesis of Theorem 1 and $\mathscr{C}$ be a shortest cycle of $G$, i.e., $\varepsilon(\mathscr{C})=g(G)$. From Lemmas 6, 7 and Theorem 4, we have the $L(G)$ is subpancyclic.

Next, we construct a family of graphs of order $n$ with $\delta'=\sqrt{2n+1}$ such that their line graphs are Hamiltonian but not pancyclic.

For any integer $d=2k+1\geqslant 3$, define the graph $G_d$ as follows. Let $\mathscr{C}=u_1u_2\ldots u_{d+1}u_1$ be a cycle of length $d+1$ and let $H_1,H_2,\ldots,H_{k+1}$ be $k+1$ copies of the empty graph of order $d-3$ such that $\mathscr{C},H_1,H_2,\ldots,H_{k+1}$ are pairwise disjoint.

Now, $G_d$ is obtained from $\mathscr{C}\cup\bigcup_{i=1}^{k+1}H_i$ by joining each vertex of $H_i$ to $u_{2i}$, for $i=1,2,\ldots,k+1$. We have

$$\delta'(G_d)=d \tag{6}$$

and

$$|V(G_d)|=(d+1)+(k+1)(d-3)=(d^2-1)/2. \tag{7}$$

Clearly, $\mathscr{C}$ is a cycle of $G$ with $\bar{\varepsilon}(\mathscr{C})=|E(G)|$, hence

$$L(G_d)\text{ is Hamiltonian.} \tag{8}$$

Obviously, $L(G_d)$ does not contain $C_d$ and hence

$$L(G_d)\text{ is not pancyclic.} \tag{9}$$

Using (6)–(9), we obtain that Theorem 1 is best possible in the sense that the condition $\delta'>\sqrt{2n+1}$ cannot be relaxed, even under the condition that $L(G)$ is Hamiltonian. This completes the proof of Theorem 1. □

**Proof of Theorem 2.** Let $G$ satisfy the hypothesis of the theorem. Assume $L(G)$ is not subpancyclic. Set $m=\max\{i\leqslant\mathrm{cr}(L(G))\,|\,L(G)\text{ does not contain }\mathscr{C}_i\}$. Then $m\leqslant\mathrm{cr}(L(G))-1$ and $L(G)$ contains $C_{m+1}$. By Theorem 4, $G$ contains a circuit $\mathscr{C}$ with $\varepsilon(\mathscr{C})\leqslant m+1<\bar{\varepsilon}(\mathscr{C})$. In fact, $\varepsilon(\mathscr{C})=m+1$, otherwise $L(G)$ contains $\mathscr{C}_m$. Using arguments similar to those in the proof of Lemma 5, we obtain

**Claim 1.** *$\mathscr{C}$ is a cycle without any chord and $\mathscr{C}$ cannot be a Hamiltonian cycle of $G$.*

Let $w$ be a vertex in $V(G)\backslash V(\mathscr{C})$. It is easy to see that

**Claim 2.** *$w$ is adjacent to at most two vertices of $\mathscr{C}$ and the distance (in $\mathscr{C}$) between any pair of $N(w)\cap V(\mathscr{C})$ is 2.*

*Otherwise $G$ contains a cycle $\mathscr{C}'$ with $\varepsilon(\mathscr{C})/2<\varepsilon(\mathscr{C}')\leqslant m$, and we obtain a contradiction as in Case* 1.1 *in Lemma* 5.

By Theorem 1, we only need to consider the case that $g(G)=4$. Let $\mathscr{C}_4$ be 4-cycle of $G$. By the arguments similar to the proof of Lemma 7, we have $\bar{\varepsilon}(\mathscr{C}_4)\geqslant 2(\delta'-2)$. By Theorem 4, $\lambda(L(G))\supseteq[3,2\delta'-4]$. Clearly,

$$\varepsilon(\mathscr{C})\geqslant(2\delta'-4)+2=2\delta'-2. \tag{10}$$

Using (10), Claims 1, 2 and $\delta'^2-\delta'>2n$, we obtain

$$\begin{aligned} n &\geqslant \left(\sum_{e\in E(\mathscr{C})}(d(e)-4)\right)\Bigg/4+\varepsilon(\mathscr{C}) \\ &= \left(\sum_{e\in E(\mathscr{C})}d(e)\right)\Bigg/4\geqslant\varepsilon(\mathscr{C})\delta'/4 \\ &\geqslant 2(\delta'-1)\delta'/4>n, \end{aligned}$$

a contradiction, which implies that $L(G)$ is subpancyclic.

Next, we construct a family of graphs of order $n$ with $\delta'^2(G)-\delta'(G)=2n$ such that their line graphs are Hamiltonian but not pancyclic.

For any integer $d=2k+1\geqslant 3$, define the graph $G_d$ as follows.

Let $\mathscr{C}=u_1,u_2,\ldots,u_{4k},u_1$ be a cycle of length $4k$ and let $H_1,H_2,\ldots,H_k$ be $k$ copies of $2k-3$ isolated vertices such that $\mathscr{C},H_1,H_2,\ldots,H_k$ are pairwise disjoint. Now $G_d$ is obtained from $\mathscr{C}\cup\bigcup_{i=1}^{k}H_i$ by joining each vertex of $H_i$ to $u_{4i-3}$ and $u_{4i-1}$, for $i=1,2,\ldots,k$. We have

$$\delta'(G)=d \tag{11}$$

and

$$|V(G_d)|=4k+k(d-4)=(d^2-d)/2. \tag{12}$$

Clearly, $\mathscr{C}$ is a cycle of $G$ with $\bar{\varepsilon}(\mathscr{C})=|E(G)|$, hence

$$L(G_d) \text{ is Hamiltonian.} \tag{13}$$

Obviously, $L(G_d)$ does not contain $\mathscr{C}_{4k-1}=\mathscr{C}_{2d-3}$ and hence

$$L(G_d) \text{ is not pancyclic.} \tag{14}$$

Using (11)–(14), we obtain that Theorem 2 is best possible in the sense that the condition $\delta'^2-\delta'>2n$ cannot be relaxed, even under the condition that $L(G)$ is Hamiltonian. This completes the proof of Theorem 2. □

From the proofs of Theorems 1 and 2, two important corollaries can easily be obtained.

**Corollary 8.** *Let G be a graph of order* $n \geqslant 72$, *girth at least* 5, *and with* $\delta' > \sqrt{2n+1}$. *Then the line graph* $L(G)$ *of* $G$ *is pancyclic whenever* $L(G)$ *is Hamiltonian, and the result is best possible.*

**Corollary 9.** *Let G be a graph of order* $n \geqslant 72$, *girth at least* 4, *and with* $\delta'^2 - \delta' > 2n$. *Then the line graph* $L(G)$ *of* $G$ *is pancyclic whenever* $L(G)$ *is Hamiltonian, and the result is best possible.*

In fact, Corollary 9 verifies the following conjecture in [5] for those graphs of order $n \geqslant 72$ with girth at least 4.

**Conjecture 10** (Liming Xiong [5]). Let $G$ be a graph with $\delta'^2 - \delta' > 2n$. Then the line graph $L(G)$ of $G$ is pancyclic whenever $L(G)$ is Hamiltonian, unless $G$ is isomorphic to $\mathscr{C}_4$, $\mathscr{C}_5$, or the Petersen graph.

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